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Discrete sedimentation model for ideal suspensions

Javier Quispe [a], Femando Concha [b], Pedro G. Toledo [a,*]

[a] Departamento de Ingeniería Química, Universidad de Concepción, Casilla 160-C, Correo 3, Concepción, Chile

[b] Departamento de Ingeniería Metalúrgica, Universidad de Concepción, Casilla 160-C, Correo 3, Concepción, Chile

Abstract

The sedimentation process of ideal suspensions is simulated using a discrete model in which gravitational, hydrodynamic, particle interaction and dispersive motions are considered as competitive processes. These mechanisms define motion rules that are implemented in regular two-dimensional lattices. Results show that the model is capable of producing the whole spectrum of particle-settling of ideal suspensions. Computer simulation results for the batch settling of rigid spheres in water are obtained for monodisperse systems. Results compared fairly well with experimental data. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

When a homogeneous suspension of solid particles and a fluid are allowed to stand in a container, the particles settle out under gravity at a rate that depends on their size, shape, weight and concentration. The dependence on concentration arises from the interaction between particles, exerted by means of the velocity distribution generated in the fluid surrounding each moving particle. The effects of size and shape of the particles take the simplest possible form when the particles are identical rigid spheres, of such small size that the Reynolds number of the fluid motion is small and inertia forces can be neglected. The mean speed of fall of a particle is then proportional to the weight and function primarily of the volume fraction of the particles. On the other hand, the sedimentation rate of one particle in a concentrated suspension is always less than the settling rate of the same particle in isolation. This is partly because the downward movement of particles causes an equal volumetric flow rate of fluid displacement relative to which the particles must move. If the particles are all uniform in a concentrated suspension, they will settle with equal velocities, apart from small statistical variations, and therefore, there will be few interparticle collisions or nearcollisions.

The sedimentation of a suspension of particles under the action of gravity has been studied extensively because of its importance in practical applications. Most of the existing models for sedimentation focus on settling rate rather than solid-concentration profiles, even though the later provides more information for characterizing sedimentation processes. This is probably due to experimental difficulties in measuring solid concentrations as a function of time.

Since the formulation of the first theory of sedimentation for suspensions of equal spheres by Kynch [1], several authors (Talmage and Fitch [2]; Shannon et al. [3,4]; Fitch, [5]) have used it to develop thickener design methods. Kynch assumed that the settling rate was a function of local solid concentration only. The assumption that the local settling velocity of solids relative to the slurry is a function only of the local solid concentration is essentially equivalent to the assumption that the only forces acting on the particles are caused by the local interstitial fluid velocity. The forces caused by fluid and solid acceleration are neglected.

The numerical simulation of the motion of particles in a fluid during sedimentation processes is a very difficult problem that until today has not been solved to entire satisfaction. Some techniques, notably finite element (Feng and Joseph [6,7]; Hu et al. [8]), finite volume, or boundary integral techniques (Wendland and Zhu, [9]), can reproduce very well the behavior of a small number of particles, but they are too computer intensive to simulate many-particle effects. Other techniques can deal with many particles, but use phenomenological expressions (Tsuji et al. [10]; Schwarzer [11]) for the coupling between particles and fluid that are incapable of rendering correctly single-particle behavior and limit severely the predictive power of a method when new parameter ranges are explored. Some techniques are valid only for small Reynolds numbers (Bossis and Brady [12]; Brady and Bossis [13]).

* Corresponding author. Fax: +56-41247491.
E-mail address: petoledo@udec.cl (P.G. Toledo).

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Recently, discrete models have been developed and successfully applied to various phenomena such as diffusion-limited aggregation of particles (Witten and Sander [14]), invasion percolation (Wilkinson and Willemsen [15]) for fluid displacement process in porous media, and sedimentation (Huang and Somasundaran [16]).

2. Model

In modeling the settling of particles we consider the following four competitive forces: gravitational, which tends to drive the particles to the bottom of the sedimentation unit; hydrodynamic or drag, function of concentration, which tends to deaden any gravitational effect; a particle interaction force, which reduces the average settling speed; and a random force which attempts to disperse the particles uniformly by dispersive motions over the whole sedimentation domain. To account for the competitive nature of these forces we introduce a parameter p representing the ratio of the probability of particle movement due to gravitation, drag and particle interaction to the probability of particle movement due to dispersive motion. Huang and Somasundaran [16] previously introduced this parameter, however forces other than gravity were not considered in their work. The main idea of incorporating p is illustrated in Fig. 1. If $p \geq 1$, downward motions predominate over Brownian motions. Notably, if $p=0$ dispersive motion rules the movement of particles and sedimentation never happens. On a square lattice, when movement is dispersive only, a particle has a probability of 1/4 to move along each of the four directions, see Fig. 1(a), but a particle can only move downward when the movement is due to a combination of gravity, drag and particle interaction, see Fig. 1(b). Fig. 1(c) represents the physical situation when all forces are present.

The model has been implemented in the computer for a square lattice. The algorithm is as follows. First, a square lattice is established in the whole domain of the sedimentation column. Second, for a given concentration, particles are randomly distributed in the sites of the lattice. Third, each particle is selected to move in one of the four directions according to a balance of probabilities; p is evaluated as a function of fluid and solid properties, concentration and degree of particle interaction. The particle is moved to a new site only if that site is empty and within the boundaries of the container. If a site is not available for the particle to move in, then the particle loses its chance of movement until the next time step. To assure equal moving opportunity for all particles, every particle is selected only once within each time unit. The procedure of select-and-move is repeated until a preset time limit is reached. The rejected movements, due to out-of-boundary or occupied destinations, are also counted for time measure.

3. Settling velocity

We consider a statistically homogeneous dispersion of identical rigid spherical particles of radius R in a Newtonian ambient fluid of viscosity μ. Inertia forces on either the solid particles or the fluid will be neglected. When a spherical particle is dropped in a viscous fluid and allowed to experience the acceleration of gravity, there is a brief transient period, after which the particle falls with a constant terminal or settling velocity. The situation is shown in Fig. 2. There are three forces acting on the particle, i.e. gravity, F_G; buoyancy, F_B; and hydrodynamic drag, F_D. The forces are collinear, so we need not be concern with their vector nature. Since the particle is moving in a straight line with no change in velocity, there is no change in linear momentum. Thus, Newton's second law reduces to the requirement that the forces on the sphere sum algebraically to zero. Gravity, buoyancy and drag forces are given by the following expressions [17],

$$\text{Gravity}: \quad F_G = \tfrac{4}{3}\rho_s g \pi R^3 \tag{1}$$

$$\text{Buoyancy}: \quad F_B = -\tfrac{4}{3}\rho_f g \pi R^3 \tag{2}$$

$$\text{Drag}: \quad F_D = -6\pi \mu R U \tag{3}$$

where ρ, is the density of the solid particle, ρ_f is the density of the fluid, U is the particle-fluid relative velocity in a

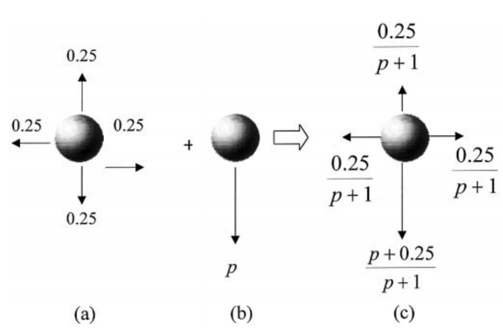

Fig. 1. Moving probabilities along various directions of a square lattice.

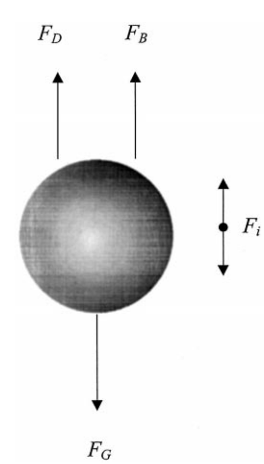

Fig. 2. Forces acting on a sphere falling in a viscous fluid.

suspension and g is the gravitational acceleration. The gravitational force is equal to the weight of the sphere and the sign is positive because it is directed downward. The buoyancy force, which points upward, is equal to the weight of the fluid displaced by the particle. The drag force, which is directed upward, represents the resistance exerted for the fluid to the movement of the particle. For an isolated particle in an infinite fluid, the balance of forces may be written as

$$F_G + F_B + F_D = 0 \tag{4}$$

In a suspension of homogeneous particles there are various other effects, some of them better understood than the others, that tend to decrease the settling rate of the particles. To account for this effect we introduce a new force, F_i, in Eq. (5). Thus, the new balance is,

$$F_G + F_B + F_D \pm F_i = 0 \tag{5}$$

We assume that this new force is just a fraction of the resulting force between gravity and buoyancy, i.e.

$$F_i = \pm\alpha\,(F_G + F_B) \tag{6}$$

with α being a parameter that can take both positive and negative values according to the probability of movement upwards or downwards. Replacing Eq. (1) to Eq. (3) and Eq. (7) into Eq. (6) gives

$$\frac{4}{3}\pi R^3\rho_s g - \frac{4}{3}\pi R^3\rho_f g - 6\pi\mu RU \pm \alpha\left(\frac{4}{3}\pi R^3\Delta\rho g\right) = 0 \tag{7}$$

Which in turn gives the following expression for the settling velocity U

$$U = \frac{\Delta\rho\,\frac{4}{3}\pi R^3 g}{6\mu\pi R}\,(1 \pm \alpha) \tag{8}$$

Accounting for the effect of concentration on the sedimentation rate is possible by virtue of Famularo and Happel [18,19]

$$\frac{U}{U_0} = \frac{1}{1 + 1.3\phi^{1/3}} \tag{9}$$

Eq. (9) applies to random assemblages of particles with

$$\Delta\rho = \rho_s - \rho = \rho_s - (\rho_s\phi - \rho_f\,(1-\phi)) \\ = (\rho_s - \rho_f)\,(1-\phi) \tag{10}$$

Thus Eq. (8) takes the form

$$U = \frac{\Delta\rho\,\frac{4}{3}\pi R^3 g\,(1-\phi)}{6\mu\pi R\left(1 + 1.3\phi^{1/3}\right)}\,(1 \pm \alpha) \tag{11}$$

In section Model we defined the parameter p as the ratio of two probabilities, i.e. the probability of particle movement due to gravitation, drag and particle interaction to the probability of particle movement due to dispersive motion. Assuming a linear relationship between probability of movement and velocity, we can write

$$p \sim \frac{U}{U_{BM}} \tag{12}$$

Where U_{BM} is the velocity of particles under dispersive motion. This velocity is constant and independent of particle size for small particles. The net effect of dispersive motion is an erratic, random motion of particles through the fluid. This effect is incorporated in the particle motion rules as Fig. 1 shows. Dispersive motion becomes important when the particle size is very small. The lower limit of p is zero, which means dispersive motion rules the movement of particles. The upper limit of p is a number big enough to reflect that the movement of particles is governed by a combination of gravity, drag and interaction forces. Based on these considerations we can write,

$$p \sim U \sim \text{weight of particle} \tag{13}$$

Eq. (11) is used to quantify the parameter p according to Eq. (13). The value of p is then used to find the movement of particles as Fig. 1 indicates.

4. Drift flux density function plot

The definition of volume average velocity of a suspension is given by Concha and Bustos [20]:

$$q = \phi U_s + (1-\phi)U_f = U_s - (1-\phi)U \tag{14}$$

where $U = U_s - U_f$ is the relative solid-fluid velocity. From Eq. (14), and since $f(\phi) = \phi U_s$ and $q = 0$ for batch settling [20],

$$f(\phi) = q\phi + \phi(1-\phi)U(\phi) \tag{15}$$

Then, the drift flux density or batch flux density function may be defined as,

$$f_b(\phi) = \phi(1-\phi)U \tag{16}$$

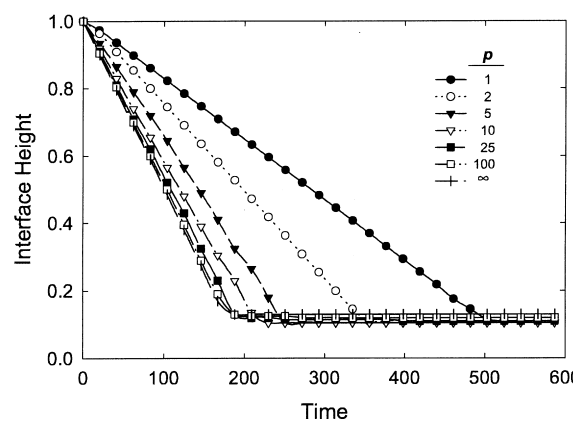

Fig. 3. Upper-interface height vs. time obtained from computer simulation.

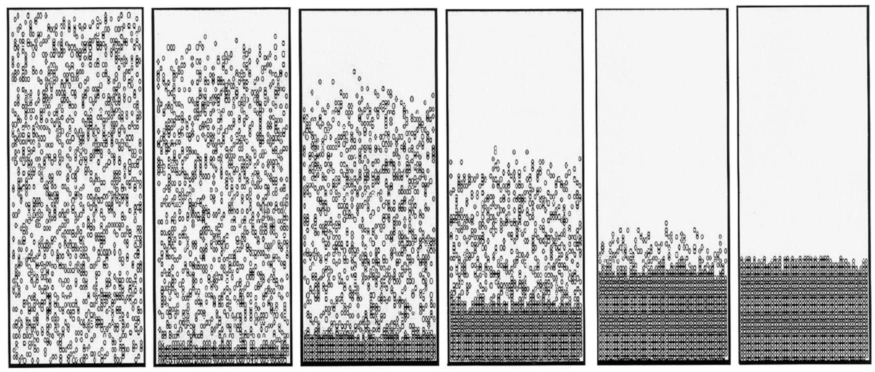

Fig. 4. Distribution of particles obtained at time=0, 50, 100, 200 and 250 for computer simulation.

5. Results and discussion

In this paper solid concentration is expressed in terms of site occupation density on a square lattice. Computer simulations were run on a 100×500 square lattice using different solids concentrations. Fig. 3 shows a quantitative comparison of settling rates. As expected, the fastest settling rate occurs when $p\to\infty$. This agrees with the fact that heavy particles falls faster than light particles. Fig. 4 shows a series of particle-distribution diagrams obtained from a typical run. It can be seen how the suspension-sediment interface grows with time as the same way that happens in a sedimentation process of monodisperse suspensions. The present model is thus successful in simulating the usual behavior of simple sedimentation systems.

Fig. 5 shows simulation results made in order to evaluate the effect of solid concentration over the settling rate. Initial volume fraction of solids, ϕ_0, ranged from 0.1 to 0.4. In this figure one can see that for dilute concentration, the settling rate is higher than for concentrated suspensions, which is expected to happen since for concentrated suspensions, particles have more probability to have collisions with their neighbors during their pathway to the bottom of the column. Otherwise, in dilute concentration, particles have more available unoccupied sites where they can move without interacting with their neighbors. The settling rate is represented by the slope of upper-interface height versus time obtained from computer simulations. To take into account the effect on one particle of the presence of all other particles, all simulations were performed with the parameter $\alpha=\pm0.1$.

Since we are working with ideal suspensions our model should reproduce the different possible types of Kynch processes for batch sedimentation, called the modes of

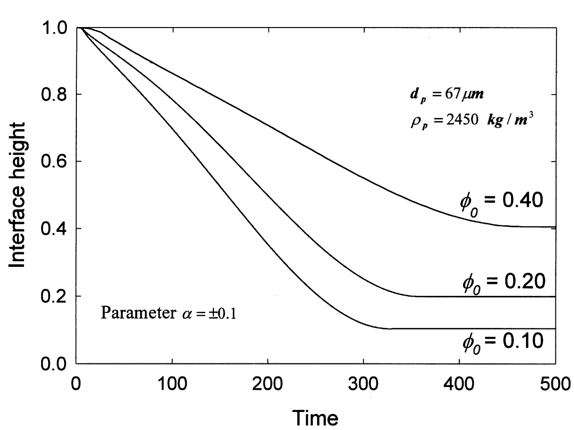

Fig. 5. Effect of particle concentration over settling rate.

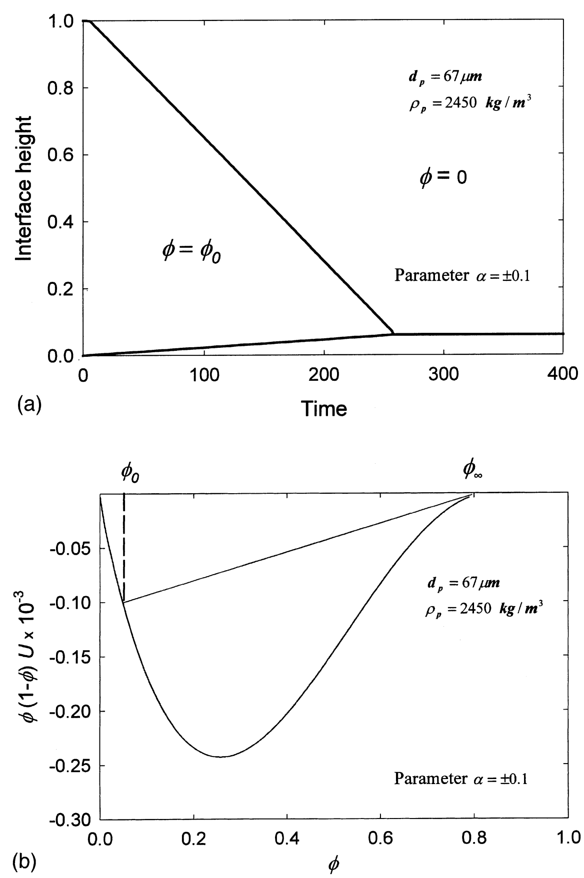

Fig. 6. (a) Settling plot for ϕ_0=0.05, a shock sedimentation mode. (b) Flux density function for a shock sedimentation mode.

sedimentation. These modes are entirely determined by the constitutive equation of the flux-density function, see Eq. (16), and initial concentration of the suspension, ϕ_0, These modes of sedimentation come from the behavior observed in the regions below the water-suspension interface in a settling plot. Independently of the modes of sedimentation, all Kynch sedimentation process consists of two regions: a suspension of concentration ϕ_∞ and a layer of clear water on top as shown in Fig. 6(a). Fig. 6(a, b) show one mode of sedimentation named shock because the concentration changes suddenly from ϕ_0 to ϕ_∞. Fig. 6(b) shows the typical flux density function for a shock where one can distinguish two points: initial concentration of suspension ϕ_0 (ϕ=0.05) from the settling plot in Fig. 6(a) and maximum possible volume fraction of the solid component, ϕ_∞, where it is possible to happen the sedimentation process.

Fig. 7(a, b) show another mode of sedimentation called contact discontinuity and a rarefaction wave. In this case, there is a continuous change in concentration from ϕ_0^* to ϕ_∞. In Fig. 7(a), one can see three zones of constant concentration plus a new one where concentration is not constant. Fig. 7(b) shows the flux density function plot for the

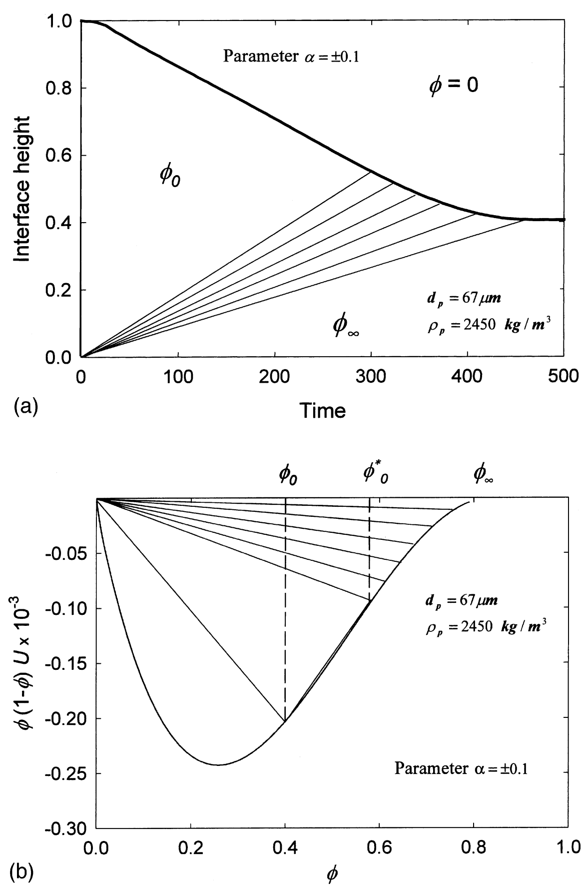

Fig. 7. (a) Settling plot for ϕ_0=0.4, a contact discontinuity and a rarefaction wave sedimentation mode. (b) Flux density function for a contact discontinuity and rarefaction wave sedimentation mode.

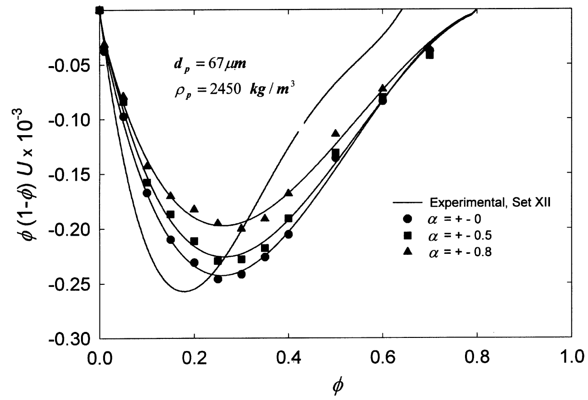

Fig. 8. Drift flux density function plot. Experimental vs. discrete model.

sedimentation process in Fig. 7(a). The nature of this mode of sedimentation is due to the concentration (ϕ=0.4).

Fig. 8 compares experimental flux data, Set XII of Shannon et al. [3], with those calculated from the simulation results of our model. To take into account the effect of parameter α over the behavior of sedimentation process, simulations were performed with different values of α. Our model captures qualitatively all features of the experimental curve.

6. Conclusions

In this paper we introduce a new discrete model for the sedimentation process of ideal suspensions. Gravity, hydrodynamic, interaction and random forces are all considered. The model within its simplicity is capable of reproducing successfully all features of the sedimentation behavior of equal size particles. The model is limited neither to fine particles nor to laminar flows. In the model particles are treated individually, thus it can be applied easily to systems with distributed particle size. Though the current simulations were performed on square lattices, they can be extended to non-lattice domains.

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